FORMATION OF GROUPS OF MUTUALLY INTERCHANGEABLE PLAYERS

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ABSTRACT

In team sports, particularly in top competition, the custom has become more frequent, almost converted into a necessity of having available on the team players that are capable of carrying out the same tasks, that is to occupy the same place on the team.

There are many circumstances that make it necessary to resort to substitutions. Among these we could mention injury, bad performance at any given moment, send off warning from the referees, tiredness, ... or simply the desire to attain regularity, in high performance, of a team that must take part in several competitions.

To be able to count on a more or less numerous group, to which to resort, convinced that the level of play will not be affected is, in our opinion, a guarantee and lack of worry for the technical team.

We are conscious of the fact that a scheme as general as this can be resolved by following several paths. We are now proposing one of these, used on many occasions for resolving different problems and of which we ourselves have made use for solving an important aspect of marketing[1]. This is not therefore a new technique although it is quite recent. What is singular about it is its use in the sphere of sports management.

Key Words

Affinity, grouping, fuzziness, uncertainty optimisation.

[1] Gil-Lafuente, J.: Marketing para el nuevo milenio. Publ.. Pirámide. Madrid 1997, pages 369-373.

Introduction to the problem

We have repeatedly insisted on the enormous importance of the concept of the fuzzy sub-set as a descriptor of an object, be this either physical or mental (of a quality, a person, of a group of people, etc.) as a consequence of its capacity for explaining complex realities that require a high degree of nuances in order to be conveniently grasped. It is for this reason that we have more than frequently resorted, in order to describe a player, to fuzzy sub-sets of the referential of his qualities, characteristics or peculiarities. We are not going to change on this occasion. Why do it if the results that we arrived at up to now have been satisfactory?

Therefore, we start out with the members of a team, competing in a determined sport, which is formed by a finite number of players, normally higher than the number taking part in a particular match. These we are going to represent by lower case letters, forming a set:

$$\{a, b, c, \dots, m\}$$

On the other hand, for exercising their sporting activities a certain number, also finite, of qualities, characteristics and peculiarities are necessary, which we also put together in a set:

$$\{A, B, C, \dots, N\}$$

Once the players on the team, and the qualities, characteristics and peculiarities, considered necessary or convenient have been specified, we proceed to "describe" each player, a, b, …, m, by means of a fuzzy sub-set of the referential of the qualities, characteristics and peculiarities A, B, …, N. In this way we arrive at as many fuzzy sub-sets as there are candidate players:

$$\begin{array}{cccccc} & A & B & & N \\ \underset{\sim}{i} = & \boxed{\mu_A^{(i)}} & \boxed{\mu_B^{(i)}} & \cdots & \boxed{\mu_N^{(i)}} \end{array}$$

$$i = a, b, c, \dots, m$$

where, as is usual $\mu_j^{(i)}$, j = A, B, …, N, represents the level at which player i possess quality, characteristic or peculiarity j.

We are going to place these fuzzy sub-sets one below the other, in such a way that on joining them they form a fuzzy matrix $[\underset{\sim}{R}]$:

A B N

$$
[R] = \begin{array}{c} a \\ b \\ \\ m \end{array}
\begin{array}{|c|c|c|c|}
\hline
\mu_A^{(a)} & \mu_B^{(a)} & \cdots & \mu_N^{(a)} \\
\hline
\mu_A^{(b)} & \mu_B^{(b)} & \cdots & \mu_N^{(b)} \\
\hline
\cdots & \cdots & \cdots & \cdots \\
\hline
\mu_A^{(m)} & \mu_B^{(m)} & \cdots & \mu_N^{(m)} \\
\hline
\end{array}
$$

In order to avoid excessive abstraction in a work that aspires to be for immediate application, we move on to show a practical example. For this we are going to consider a Sporting Limited Company that wishes to homogeneously group its players with a view to eventual interchangeability. So as not to draw out the operative aspect of this problem, we will limit ourselves to 10 players and 4 qualities, on the understanding that the extension to any finite number of one or the other does not alter, in the very least, the validity of the model.

The following then will be the fuzzy descriptor sub-sets for the players:

	A	B	C	D
a =	.8	.4	1	.6

	A	B	C	D
b =	.3	1	.7	1

	A	B	C	D
c =	.5	.9	.6	.9

	A	B	C	D
d =	.9	.6	.5	.7

	A	B	C	D
e =	.2	.7	.9	.5

	A	B	C	D
f =	1	.8	.9	.6

	A	B	C	D
g =	.4	.8	.9	.7

	A	B	C	D
h =	.6	.5	.8	1

	A	B	C	D
l =	.7	.9	.9	.5

	A	B	C	D
q =	.1	.9	.9	.8

From this information we can immediately form a fuzzy matrix[R] in which the rows will describe the skills of each one of the players and the columns the level at which these qualities, characteristics or peculiarities are possessed by each one of the players under consideration.

[R] =

	A	B	C	D
a	.8	.4	1	.6
b	.3	1	.7	1
c	.5	.9	.6	.9
d	.9	.6	.5	.6
e	.2	.7	.9	.5
f	1	.8	.9	.6
g	.4	.8	.9	.7

h	.6	.5	.8	1
l	.7	.9	.9	.5
q	.1	.9	.9	.8

Establishment of sufficient levels in the relative skills

Once this information has been arrived at the need arises to take an important decision relative to the degree of homogeneity desired for each quality, characteristic and peculiarity, In fact, if we take as an example a sport such as football and a skill such as "ball control", it is obvious that from perfect control (degree one) to the total lack of this quality (degree 0) there is almost an infinite range of levels. The problem here means establishing to what degree ball control is considered as good for a player and, therefore, it is accepted that he possess this quality. If a decision is taken to the effect that to degree 0.8 is sufficient, then all players possessing this quality at level 0.8 or higher will be accepted as good (that is if they possess it) in the ball controlling skill.

What we have just stated is equivalent, in the language of mathematics for uncertainty, to establishing for each quality, characteristic and peculiarity, a threshold in [0, 1] which it is customary to designate with the Greek letter θ. Therefore, for each skill a threshold is imposed and in this way there now exist as many thresholds as there are elements of the referential of qualities, characteristics and peculiarities. Generally speaking we will have:

$$0 \leq \theta_j \leq 1 \ , \ j = A, B, \dots, N$$

Once the values of θ_j, $j = A, B, \dots, N$, the valuations of each column are compared with their corresponding threshold. If the valuation is equal to or higher than the threshold then it is considered that the player possess the reference skill, and in a new matrix a one is placed in the corresponding box. If the opposite is true then a zero is placed in that box. This can be written as:

$$\begin{array}{llll} \mu_j^{(i)} \geq \theta_j \quad , & \beta_j^{(i)} = 1 & & \\ \mu_j^{(i)} < \theta_j \quad , & \beta_j^{(i)} = 0 & , & i = a, b, \dots, m \\ & & & j = A, B. \dots, N \end{array}$$

With this a new matrix is arrived at in which the boxes will only contain ones or zeros. With the object of arriving at a clearer presentation it is normal to omit the zeros from the matrix by leaving the box empty. It will not have escaped the reader's notice that this is a boolean matrix, which we shall designate [B]. In general it can be shown as follows:

$$[B] = \begin{array}{c|cccc} & A & B & \cdots & N \\ \hline a & \beta_A^{(a)} & \beta_B^{(a)} & \cdots & \beta_N^{(a)} \\ b & \beta_A^{(b)} & \beta_B^{(b)} & \cdots & \beta_N^{(b)} \\ & \cdots & \cdots & \cdots & \cdots \\ m & \beta_A^{(m)} & \beta_B^{(m)} & \cdots & \beta_N^{(m)} \end{array}$$

$\beta_j^{(i)} \in \{0, 1\}$

$i = a, b, \ldots, m$

$j = A, B. \ldots, N$

If in our case we consider the following as the thresholds:

$$\theta_A = 0.7 \; , \theta_B = 0.8 \; , \theta_C = 0.9 \; , \theta_D = 0.7$$

the fuzzy matrix or relation [R] becomes the following boolean matrix:

[B] =

	A	B	C	D
a	1		1	
b		1		1
c		1		1
d	1			
e			1	
f	1	1	1	
g		1	1	1
h				1
l	1	1	1	
q		1	1	1

This matrix can be the reference so that by means of adequate treatment, we arrive at the sought after objective that consists in the homogeneous grouping of players who are capable of mutual substitution or interchangeability. To these effects we have available a theory from which certain algorithms have been drawn up, the use of which have provided excellent results in different areas of the economic and business field. We are referring to the theory of affinities[2].

Algorithm for arriving at affinities

We are now going to reproduce the steps that make up one of the algorithms that are very easy to use and it is known as "the algorithm of maximum inverse correspondence"[3].

1) Selection from among a set of players and a set of qualities, characteristics and peculiarities, the one possessing the least number of elements.
2) From the set with the least number of elements its "power set" is constructed, that is the set of all its parts (most powerful set).
3) For each element of the "power set" the elements of the set that was not selected as it had more elements is made to correspond. This is then called the "connection to the right".
4) We now choose for each set that is not empty of the "connection to the right" the corresponding one from the "power set" that possesses the greatest number of elements.
5) The relations between the two sets that have been arrived at form aGalois lattice, which, apart from showing the different homogeneous groupings, allows us to carry out a perfect structuring of the same.

Applying this algorithm to our case brings up no special problems. Let us take a look at it:

1) The set with the least number of elements is that of the qualities, characteristics and peculiarities.

{A, B, C, D}

2) In our case the "power set" will be:

{∅, A, B, C, D, AB, AC, AD, BC, BD, CD, ABC, ABD, ACD, BCD, ABCD}

Notice that the "power set" includes all the possible combinations of qualities, characteristics and peculiarities, taken one by one, two by two, three by three and the

[2] The theory of affinities has its origin in a work by Professors Kaufmann, A. and Gil Aluja, J.: "Selection of affinities by means of fuzzy relations and Galois lattices". Proceedings of the IX Euro Congress O.R. Aachen, 16-19 July 1991.

[3] Gil-Aluja, J.: Elements for a theory of decision in uncertainty. Kluwer Academic Publishers. Boston, London, Dordrecht, 1999, pages. 215.

combination of four. In this way the possession by a player of a combination that is not in the "power set" will not occur.

3) We now place the elements of the "power set" in the form of a column to the left, placing another column to the right of the previous one making every possible combination of qualities, characteristics and peculiarities correspond.

In order to arrive at these relations it is sufficient to look at matrix [B] and find the boxes in which there are ones, first one by one, then two by two, later three by three and finally to see if there is any row in all the columns where there are ones. Let us take a detailed look at our example.

In column A there are ones in boxes a, d, f, l, which indicates that quality, characteristic or peculiarity A is possessed by players a, d, f, l. In column B there are ones in boxes b, c, f, g, l, q. These then are the players possessing quality, characteristic or peculiarity B. …. The same is done for columns C and D.

We now look at columns A and B simultaneously and it will be seen that there are only ones in the <u>two boxes</u> in boxes f, l. This brings to light the fact that only players f, l posses these two qualities, characteristics or peculiarities A and B. We then continue on with the process and look at columns A and C, and see that there are ones in the two boxes of rows a, f, l which tells us that these players possess qualities, characteristics or peculiarities A and C. An so on successively.

Now we take a look at columns A, B, and C and see that there are only ones, in the three boxes of rows f, l. These then are the players possessing the three qualities, characteristics or peculiarities

After studying what happens simultaneously in the columns for the elements of the "power set" we will have arrived at the following correspondences:

$\varnothing \rightarrow$ abcdefghlq
A $\rightarrow$ adfl
B $\rightarrow$ bcfglq
C $\rightarrow$ aefglq
D $\rightarrow$ bcghq
AB $\rightarrow$ fl
AC $\rightarrow$ afl
AD $\rightarrow \varnothing$
BC $\rightarrow$ fglq
BD $\rightarrow$ bcgq
CD $\rightarrow$ gq

$$\begin{aligned} ABC &\rightarrow fl \\ ABD &\rightarrow \varnothing \\ ACD &\rightarrow \varnothing \\ BCD &\rightarrow gq \\ ABCD &\rightarrow \varnothing \end{aligned}$$

We should remember that, relative to the above, the first column represents all the possible groupings of qualities, characteristics or peculiarities and the second the groups of players possessing the level required by the threshold.

A quick glance will tell us that there are no players at all possessing qualities, characteristics or peculiarities AD (this relation has been shown with the vacant sign, $\varnothing$). Therefore there will be no player possessing simultaneously ABD, ACD nor A, B, C, D, (again the vacant symbol $\varnothing$ has been used for these correspondences). These correspondences therefore must be initially separated from the process, although later on the last will be recuperated in order to give coherence to the graphic representation by means of a lattice.

Now let us concentrate on the second column, and here it will be seen that there are certain players that have correspondences with more than one grouping of qualities, characteristics or peculiarities (first column). This occurs in our case with players g, q, who both possess qualities CD and also BCD. Therefore as BCD includes CD, the correspondence with less qualities, characteristics or peculiarities is eliminated (lees upper case letters). The same occurs with players f, l and skills AB and ABC, where we eliminate the correspondence $AB \rightarrow fl$. Once we have completed these eliminations we are left with the maximum correspondences, in the sense that for each group of players we have the greatest number of skills that they are capable of carrying out. For showing this the custom has been acquired of inverting the columns by placing, in this case, to the left the homogenous groups of players and to the right the qualities, characteristics or peculiarities they possess. In this case we arrive at:

$$\begin{aligned} \varnothing &\rightarrow ABCD \\ fl &\rightarrow ABC \\ gq &\rightarrow BCD \\ afl &\rightarrow AC \\ bcgq &\rightarrow BD \\ fglq &\rightarrow BC \end{aligned}$$

$$\begin{aligned} adfl &\rightarrow A \\ bcghq &\rightarrow D \\ bcfglq &\rightarrow B \\ aefglq &\rightarrow C \\ abcdefghlq &\rightarrow \varnothing \end{aligned}$$

The name of "affinities" has been give to these maximum relations, which bring to light, without a single doubt, the largest possible groupings of players that are capable of carrying out, at the required level, the highest number of skills that are common to them all.

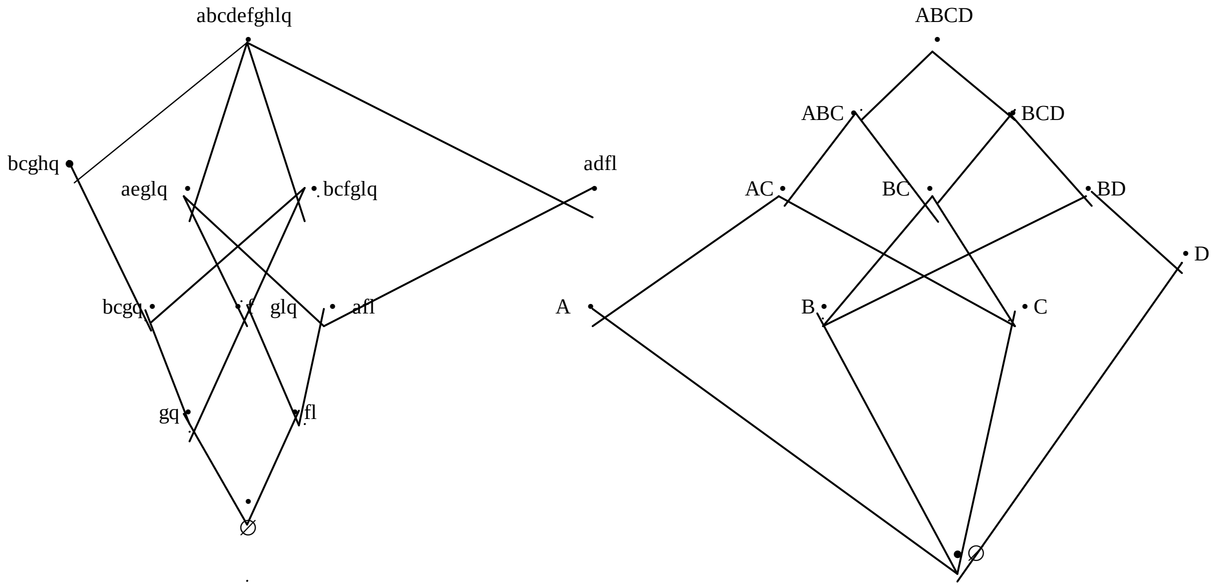
5) Professors Kaufmann y Gil-Aluja, in several of their works[4] consider that, in spite of having arrived at the sought after affinities, it is possible to improve their presentation by structuring them in an ordered manner, in this way making the decision taking easier. We should not forget that the objective we are seeking is to get to know which of the available players in a Sporting Company or Club are interchangeable with regard to their qualities, characteristics or peculiarities that are required at any given moment or during a determined phase of the sporting activity. For this they proposed "organising" the affinities by resorting to aGalois Lattice.

But let us take this step by step. If only the grouping of players (first column) are considered these can be placed as vertices of a lattice that is formed starting out from a vertex placed at the lower extreme to which a vacancy $\varnothing$ is assigned. From this then draw as many arcs as there are groups of players in which there are no common players (fl, gq). These two groupings are assigned to the final vertices of those arcs, We now draw as many arcs starting out from each vertex as there are groups of players, formed by <u>the same</u> players plus another. And so on successively. The lattice will be closed with a group that includes all the players.

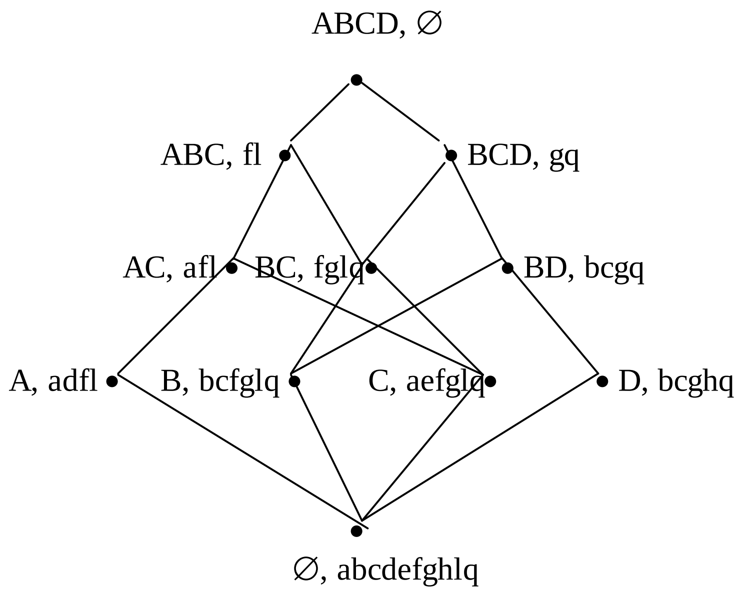
The same can be done with the groupings of qualities, characteristics and peculiarities (second column). On acting in this way it will be seen that the lattice is

[4] Kaufmann, A. y Gil-Aluja, J.: Técnicas de gestión de empresa. Previsiones, decisiones y estrategias. Publ. Pirámide. Madrid 1992,pages. 398-401.
Kaufmann, A. y Gil-Aluja, J.: Técnicas especiales para la gestión de Expertos.Publ. Milladoiro. Santiago de Compostela 1993, pages. 161-165.
Gil-Aluja, J.:Elements for a theory of decision in uncertainty. Kluwer Academic Publishers. London, Boston, Dordrecht 1999, pág. 212-215.

the same but is inverted. What we have therefore are two isomorphic lattices. Let us take a look at this.



If one of the lattices is rotated 90º and then superimposed, we can verify the isomorphism and we can observe the affinities between players and characteristics that they possess in common. This lattice then is aGalois lattice.



Arriving at this Galois lattice finalises the algorithm. Now what remains is to study the same and later take decisions that are suitable to the sporting requirements at the time.

Contents and significance of the Galois lattice

The Galois lattice not only shows us in an ordered and symmetrical way the total number of affinities that exist between the players, listing the skills possessed by each homogeneous group of the same, but it also interrelates the same by means of a coherent structure.

In fact, as can be easily seen in our case, if we start out from the vertex of the lowest part of the lattice, as we move upwards, in each group of players the number of qualities, characteristics or peculiarities possessed by the members of the different groups increases and the number of players decreases. Thus, the first vertex indicates that if we wanted to form a group with all the players, there is no single skill possessed by them all. But if we are willing to consider a single quality, characteristic or peculiarity, then certain players that possess the same appear, and in this way form homogeneous groups relative to this particular skill.Thus quality, characteristic or peculiarity A is possessed by the group of players formed by a, d, f, l; skill B, by the group made up of players b, c, f, g, l, q; …… When we reach the point where we need to take into account two qualities, characteristics or peculiarities the group of players becomes even more reduced In this way in our case it will be seen that skills are only possessed by the group of players a, f, l. Pay special attention to this case: On adding to A quality, characteristic or peculiarity C we have lost playerd from the group of players possessing the former; and likewise on adding to C quality, characteristic or peculiarity A players e, g, q. have been eliminated from the previous group. And so on as we move up the lattice vertically we gain more skills and loose players, up to the point in which on requiring all the qualities, characteristics or peculiarities the group of players is reduced to an absolute minimum which, in our case, is the total non-existence of the same, that is the group is "vacant".

Taken from another angle, if the analysis is initiated from the top towards the bottom, it will be seen that then upper vertex of the lattice tells us on requiring all the qualities, characteristics or peculiarities that no player exists possessing the all. But if we waive one or more of these we now find certain groups that do posses them. In fact if we do not consider skill D, there are two players to form a group: f, l. In fact if skill D is not considered, two players already form a group: f, l If A is not considered, then players g, q make up the group that possess qualities, characteristics or peculiarities B, C, D. I from the group with skills ABC we do away with Ba will be added to the previous group of players, whilst if we eliminate skill A the players joining the previous group are g, q forming a new group f, g, l, q. In this way, as we eliminate qualities, characteristics or peculiarities, the groups of players becomes more numerous. As we minimise the requirement for skills the group reaches its maximum number. In our case it can be said (more from a formal than from a pragmatic point of view) that no quality, characteristic or peculiarity is possessed by all the players.

Having made this short incursion into the "reading" of the lattice, we are now going to delve into the field of reality, by placing ourselves in a situation which ever more frequently is faced by sports management. Throughout a competition, a place on the team has been filled by a player beginning to show signs of tiredness with the consequent decrease in his level of play. In order to cover this position well, certain skills are required. Which player on the team can substitute the up to now main player with certain guarantees of success? To put it even simpler. Throughout a match a player has not yielded sufficiently or suffers an accident. Which of his teammates can substitute for him? Even further, As a consequence of the way the match has been going, the strategy must be changed. For this a player must go onto the filed with different qualities, characteristics or peculiarities. Given the new requirements, which players can carry out the new task? An so on, we could continue to show new situations, the solution to which are practically automatic, if we have the affinities available, particularly of shown by means of a Galois lattice.

Although this may seem superfluous, we cannot resist the temptation to complete what we have just stated from a general point of view, with an application to the case we have presented. Therefore, in the even the technical director were to consider it necessary to have skills B, D, for example, then he can resort to four players b, c, g, q. If he is happy with only one, D for example, then he has one more player available, h. But in the event that the circumstances or incidents in the match were to change, and now the qualities, characteristics or peculiarities required are A, C, for example, he will have to use the three players that are capable of carrying them out in the field, and these are a, f, l. In this way the whole map of possible decisions can be seen by the technical director. And this without any error or omission.

Conclusions

The scheme we have proposed is aimed at an eminently practical point of view, for immediate use in the most diverse realities that normally crop up in the daily events of Sporting Companies or Clubs the activity of which takes place in what are well known as team sports. This of course, does not prevent its being used in the sphere of individual sports, with the small modifications this would entail. The method is sufficiently flexible and adaptive so that this transfer would not cause any great (or even small) problems.

The objective of "practicality" advises supporting the study on an algorithm of which we have avoided its theoretical justifications. which on the other hand are sufficiently well known by theoreticians in the sphere of management operating techniques[5]. Merely as an indication allow us to point out the enormous interest acquired, in this aspect, by the concept of the Moore family, Moore closing and Moore closures[6]. We hope that in spite of this our work has not lost coherence and that the line we have followed has permitted easy comprehension of the process.

From an operational point of view we may well ask ourselves what happens when a high number of qualities, characteristics and peculiarities are taken into account as significant, at the same time as there is a large number of players to be grouped. In a case like this, on initiating the algorithm we will be faced with a "power set" with a large quantity of elements. The combinatory process makes it difficult to continue with the following steps of the algorithm, <u>if we follow a manual process</u>. This latter part of the previous sentence provides the solution. Since the development has been based on an algorithm, its incorporation into data processing is immediate. In this way, by simply pressing a button on the key board the sought after solution is arrived at automatically. Any adaptations to changes that, obviously take place as a consequence of the state of play have an immediate answer.

Finally, we should like to state that all we have presented above, has no intention of being a neither finished nor completed work. It is more like an attempt to open a door to the study of a whole range of problems that reality frequently, in an area, such as the world of sport, that all the time is acquiring greater importance in the social, economic, and we would even dare to say in the cultural environment.

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